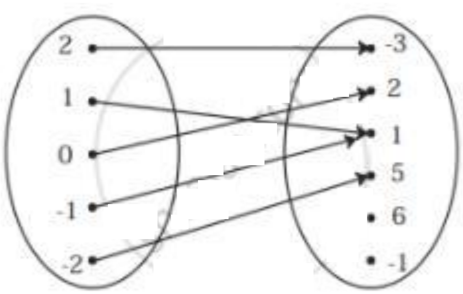
Chapter 2: Functions

EXERCISE 2.1 [PAGES 30 - 31]

Exercise 2.1 | Q 1.1 | Page 30

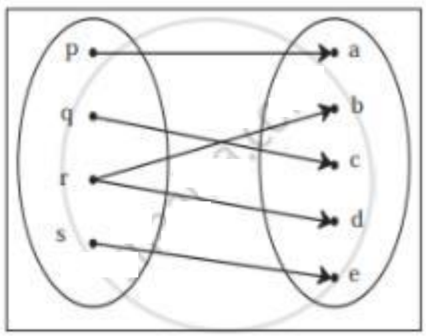
Check if the following relation is function.



SOLUTION

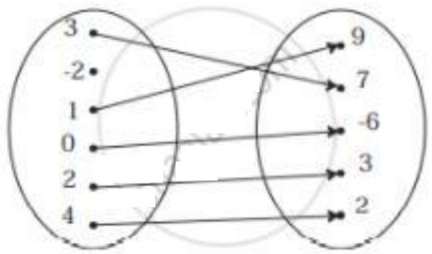
Yes
Reason:
Every element of set A has been assigned a unique element in set B.

Exercise 2.1 | Q 1.2 | Page 31
Check if the following relation is function.



No.
Reason:
An element of set A has been assigned more than one element from set B.

Exercise 2.1 | Q 1.3 | Page 31
Check if the following relation is function.



SOLUTION

No.
Reason:
Not every element of set A has been assigned an image from set B.

Exercise 2.1 | Q 2.1 | Page 31
Which sets of ordered pairs represent functions from $A = \{1,2,3,4\}$ to $B = \{-1,0,1,2,3\}$? Justify
$\{(1,0), (3,3), (2,-1), (4,1), (2,2)\}$

SOLUTION

$\{(1, 0), (3, 3), (2, -1), (4, 1), (2, 2)\}$ does not represent a function.
Reason:
$(2, -1)$ and $(2, 2)$ show that element $2 \in A$ has been assigned two images – 1 and 2 from set B.

Exercise 2.1 | Q 2.2 | Page 31
Which sets of ordered pairs represent functions from $A = \{1,2,3,4\}$ to $B = \{-1,0,1,2,3\}$? Justify
$\{(1,2), (2,-1), (3,1), (4,3)\}$

SOLUTION

$\{(1, 2), (2, -1), (3, 1), (4, 3)\}$ represents a function. **Reason:**
Every element of set A has a unique image in set B.

Exercise 2.1 | Q 2.3 | Page 31
Which set of ordered pair represent function from $A = \{1,2,3,4\}$ to $B = \{-1,0,1,2,3\}$? Justify.
$\{(1,3), (4,1), (2,2)\}$

SOLUTION

$\{(1, 3), (4, 1), (2, 2)\}$ does not represent a function.
Reason:
$3 \in A$ does not have an image in set B.

Exercise 2.1 | Q 2.4 | Page 31

Which set of ordered pair represent function from
A = {1,2,3,4}to B = {−1,0,1,2,3}? Justify
{(1,1), (2,1), (3,1), (4,1)}

SOLUTION

{(1, 1), (2, 1), (3, 1), (4, 1)} represents a function **Reason:**
Every element of set A has been assigned a unique image in set B.

Exercise 2.1 | Q 3.1 | Page 31
If f(m) = $m^2 - 3m + 1$, find f(0)

SOLUTION

f(m) = $m^2 – 3m +1$
f(0) = $0^2 – 3(0) + 1 = 1$

Exercise 2.1 | Q 3.2 | Page 31
If f(m) = $m^2 - 3m + 1$, find f(−3)

SOLUTION

f(–3) = $(–3)^2 – 3(–3) + 1$
= 9 + 9 + 1 = 19

Exercise 2.1 | Q 3.3 | Page 31

If f(m) = $m^2 - 3m + 1$, find $f\left(\frac{1}{2}\right)$

SOLUTION

$$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 = 3\left(\frac{1}{2}\right) + 1 = \frac{1}{4} = \frac{3}{2} + 1$$
$$= \frac{1-6+4}{4} = -\frac{1}{4}$$

Exercise 2.1 | Q 3.4 | Page 31
If f(m) = $m^2 - 3m + 1$, find f(x + 1)

SOLUTION

f(x + 1) = $(x + 1)^2 – 3(x + 1) + 1$
= $x^2 + 2x + 1 – 3x – 3 + 1$
= $x^2 – x – 1$

Exercise 2.1 | Q 3.5 | Page 31
If f(m) = m2 − 3m + 1, find f(−x)

SOLUTION

$f(-x) = (-x)^2 - 3(-x) + 1$
$= x^2 + 3x + 1$

Exercise 2.1 | Q 4.1 | Page 31

Find x, if g(x) = 0 where

$g(x) = \frac{5x-6}{7}$

SOLUTION

$g(x) = \frac{5x-6}{7}$

$g(x) = 0$

$\therefore \frac{5x-6}{7} = 0$

$\therefore 5x - 6 = 0$

$\therefore x = \frac{6}{5}$

Exercise 2.1 | Q 4.2 | Page 31

Find x, if g(x) = 0 where

$g(x) = \frac{18-2x^2}{7}$

SOLUTION

$g(x) = \frac{18-2x^2}{7}$

$g(x) = 0$

$\therefore \frac{18-2x^2}{7} = 0$

$\therefore 18 - 2x^2 = 0$

$\therefore x^2 = \frac{18}{2} = 9$

$\therefore x = \pm 3$

Exercise 2.1 | Q 4.3 | Page 31
Find x, if g(x) = 0 where
$g(x) = 6x^2 + x - 2$

SOLUTION

$g(x) = 6x^2 + x - 2$

$g(x) = 0$

$\therefore 6x^2 + x - 2 = 0$

$\therefore 6x^2 + 4x - 3x - 2 = 0$

$\therefore 2x(3x + 2) - 1(3x + 2) = 0$

$\therefore (2x - 1)(3x + 2) = 0$

$\therefore 2x - 1 = 0$ or $3x + 2 = 0$

$\therefore x = \frac{1}{2}$ or $x = -\frac{2}{3}$

Exercise 2.1 | Q 5 | Page 31
Find x, if $f(x) = g(x)$ where
$f(x) = x^4 + 2x^2$, $g(x) = 11x^2$

SOLUTION

$f(x) = x^4 + 2x^2$, $g(x) = 11x^2$

$f(x) = g(x)$

$\therefore x^4 + 2x^2 = 11x^2$

$\therefore x^4 - 9x^2 = 0$

$\therefore x^2(x^2 - 9) = 0$

$\therefore x = 0$ or $x^2 - 9 = 0$

$\therefore x = 0$ or $x^2 = 9$

$\therefore x = 0$ or $x = \pm 3$

Exercise 2.1 | Q 6 | Page 31
If (x) ={x^2 + 3, x≤2, 5x+ 7, x>2 , then find
f(3)

f(2)
f(0)

SOLUTION

$x^2 + 3$, $x \le 2$, $5x + 7$, $x > 2$

i. $f(3) = 5(3) + 7 = 15 + 7 = 22$

ii. $f(2) = 2^2 + 3 = 4 + 3 = 7$

iii. $f(0) = 0^2 + 3 = 3$

Exercise 2.1 | Q 7 | Page 31

If $f(x) = \{ 4x - 2$,
$x \le -3$ 5,
$-3 < x < 3$,
x^2,
$x \ge 3$ then find $f(-4)$, $f(-3)$, $f(1)$, $f(5)$

SOLUTION

$f(x) = 4x - 2$,
$x \le -3$ 5,
$-3 < x < 3$,
x^2,
$x \ge 3$

i. $f(-4) = 4(-4) - 2 = -16 - 2 = -18$

ii. $f(-3) = 4(-3) - 2 = -12 - 2 = -14$

iii. $f(1) = 5$

iv. $f(5) = 5^2 = 25$

Exercise 2.1 | Q 8.1 | Page 31

If $f(x) = 3x + 5$, $g(x) = 6x - 1$, then find $(f+g)(x)$

SOLUTION

$f(x) = 3x + 5$, $g(x) = 6x - 1$

$(f+g)x = f(x) + g(x)$

$= 3x + 5 + 6x - 1 = 9x + 4$

Exercise 2.1 | Q 8.2 | Page 31

If $f(x) = 3x + 5$, $g(x) = 6x - 1$, then find $(f - g)(2)$

SOLUTION

$(f - g)(2) = f(2) - g(2)$

= [3 (2) + 5] – [6 (2) – 1]

= 6 + 5 – 12 + 1= 0

Exercise 2.1 | Q 8.3 | Page 31

If $f(x) = 3x + 5$, $g(x) = 6x – 1$, then find (f g) (3)

SOLUTION

$(f\ g)(3) = f(3)\ g(3)$

= [3 (3) + 5] [6 (3) – 1]

= (14) (17) = 238

Exercise 2.1 | Q 8.4 | Page 31

If f(x) = 3x + 5, g(x) = 6x – 1, then find $\left(\frac{f}{g}\right)(x)$ and its domain

SOLUTION

$$\left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)} = \frac{3x+5}{6x-1}, x \neq \frac{1}{6}$$

Domain = R - $\left\{\frac{1}{6}\right\}$

Exercise 2.1 | Q 9.1 | Page 31

If $f(x) = 2x^2 + 3$, $g(x) = 5x - 2$, then find $f \circ g$

SOLUTION

$f(x) = 2x^2 + 3$, $g(x) = 5x – 2$

$(fog)(x) = f(g(x)) = f(5x – 2)$

$= 2(5x – 2)^2 + 3$

$= 2(25x^2 – 20x + 4) + 3$

$= 50x^2 – 40x + 8 + 3$

$= 50x^2 – 40x + 11$

Exercise 2.1 | Q 9.2 | Page 31

If $f(x) = 2x^2 + 3$, $g(x) = 5x – 2$, then find g of

SOLUTION

$(g\ of)(x) = g(f(x)) = g(2x^2 + 3)$

$= 5(2x^2 + 3) – 2$

$= 10\ x^2 + 15 – 2$

$= 10\ x^2 + 13$

NOTES

$(gof)\ (x) = g(f\ (x)) = g(2x^2 + 3)$

$= 5(2x^2 + 3) – 2$

$= 10\ x\ 2 + 15 – 2$

$= 10\ x^{2} + 13$

Exercise 2.1 | Q 9.3 | Page 31

If $f(x) = 2x^2 + 3$, $g(x) = 5x – 2$, then find fof

SOLUTION

$(fof)\ (x) = f(f(x)) = f(2x^2 + 3)$

$= 2(2x^2 + 3)^2 + 3$

$= 2\ (4x^4 + 12x^2 + 9) + 3$

$= 8x^4 + 24x^2 + 18 + 3$

$= 8x^4 + 24x^2 + 21$

Exercise 2.1 | Q 9.4 | Page 31

If $f(x) = 2x^2 + 3$, $g(x) = 5x – 2$, then find gog

SOLUTION

$(gog)\ (x) = g\ (g\ (x)) = g\ (5x – 2)$

$= 5(5x – 2) – 2$

$= 25x – 10 – 2$

$= 25x – 12$

MISCELLANEOUS EXERCISE 2 [PAGE 32]

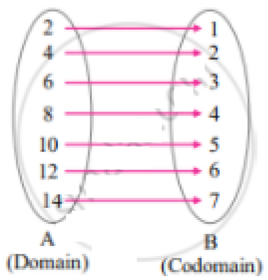
Miscellaneous Exercise 2 | Q 1.1 | Page 32

Which of the following relations are functions? If it is a function determine its domain and range.

{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5), (12, 6), (14, 7)}

SOLUTION

{(2, 1), (4, 2), (6, 3), (8, 4), (10, 5) (12, 6), (14, 7)}



Every element of set A has been assigned a unique element in set B.

∴ Given relation is a function.
Domain = {2, 4, 6, 8, 10, 12, 14},
Range = {1, 2, 3, 4, 5, 6, 7}

Miscellaneous Exercise 2 | Q 1.2 | Page 32
Which of the following relations are functions? If it is a function determine its domain and range.
{(0, 0), (1, 1), (1, −1), (4, 2), (4, −2), (9, 3), (9, −3), (16, 4), (16, -4)}

SOLUTION

{(0, 0), (1, 1), (1, –1), (4, 2), (4, –2), (9, 3), (9, –3), (16, 4), (16, – 4)}

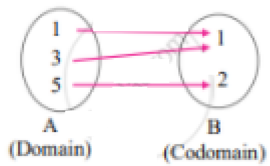
∵ (1, 1), (1, –1) ∈ the relation

∴ Given relation is not a function. As the element 1 of the domain has not been assigned a unique element of co-domain.

Miscellaneous Exercise 2 | Q 1.3 | Page 32
Which of the following relations are functions? If it is a function determine its domain and range.
{(1, 1), (3, 1), (5, 2)}

SOLUTION

{(1, 1), (3, 1), (5, 2)}



Every element of set A has been assigned a unique element in set B.

∴ Given relation is a function.
Domain = {1, 3, 5}, Range = {1, 2}

Miscellaneous Exercise 2 | Q 2 | Page 32

A function f: R→R defined by f(x) = $3\frac{x}{5}+2,$ x∈R. Show that f is one-one and onto. Hence find f^{-1}

SOLUTION

f: R →R defined by f(x) = $\frac{3x}{5}$ +2

First we have to prove that f is one-one function for that we have to prove if $f(x_1) = f(x_2)$ then $x_1 = x_2$

Here f(x) = $\frac{3x}{5}+2$

Let f(x1) = f(x2)

$$\therefore \frac{3x_1}{5}+2=\frac{3x_2}{5}+2$$

$$\therefore \frac{3x_1}{5}=\frac{3x_2}{5}$$

∴ $x_1 = x_2$

∴ f is a one-one function. Now, we have to prove that f is an onto function. Let y ∈ R be such that y = f(x)

∴ y = $\frac{3x}{5}+2$

∴ y -2 = $\frac{3x}{5}$

$$\therefore x=\frac{5(y-2)}{3}\in R$$

∴ for any y ∈ co-domain R, there exist an element x = $\frac{5(y-2)}{3}$ domain R such that f(x) = y

∴ f is an onto function.

∴ f is one-one onto function.

$\therefore f^{-1}$ exists

$$\therefore f^{-1}(y) = \frac{5(y-2)}{3}$$

$$\therefore f^{-1}(x) = \frac{5(x-2)}{3}$$

Miscellaneous Exercise 2 | Q 3 | Page 32

A function f is defined as follows

$f(x) = 4x + 5$, for $-4 \le x < 0$.

Find the values of $f(-1)$, $f(-2)$, $f(0)$, if they exist.

SOLUTION

$f(x) = 4x + 5, -4 \le x < 0$

$f(-1) = 4(-1) + 5 = -4 + 5 = 1$

$f(-2) = 4(-2) + 5 = -8 + 5 = -3$

$x = 0 \notin$ domain of f

$\therefore$ f(0) does not exist.

Miscellaneous Exercise 2 | Q 4 | Page 32

A function f is defined as follows: $f(x) = 5 - x$ for $0 \le x \le 4$ Find the value of x such that $f(x) = 3$

SOLUTION

$f(x) = 5 - x$

$f(x) = 3$

$\therefore 5 - x = 3$

$\therefore x = 5 - 3 = 2$

Miscellaneous Exercise 2 | Q 5 | Page 32

If $f(x) = 3x^2 - 5x + 7$ find $f(x - 1)$.

SOLUTION

$f(x) = 3x^2 - 5x + 7$

$\therefore f(x - 1) = 3(x - 1)^2 - 5(x - 1) + 7$

$= 3(x^2 - 2x + 1) - 5(x - 1) + 7$

$= 3x^2 - 6x + 3 - 5x + 5 + 7$

$= 3x^2 - 11x + 15$

Miscellaneous Exercise 2 | Q 6 | Page 32
If f(x) = 3x + a and f(1) = 7 find a and f(4).

SOLUTION

f(x) = 3x + a

f(1) = 7

∴ 3(1) + a = 7

∴ a = 7 – 3 = 4

∴ f(x) = 3x + 4

∴ f(4) = 3(4) + 4 = 12 + 4 = 16

Miscellaneous Exercise 2 | Q 7 | Page 32
If f(x) = ax^2 + bx + 2 and f(1) = 3, f(4) = 42. find a and b.

SOLUTION

f(x) = ax^2 + bx + 2

f(1) = 3

∴ $a(1)^2$ + b(1) + 2 = 3

∴ a + b = 1 …(i)

∴ f(4) = 42

∴ a(4)2 + b(4) + 2 = 42

∴ 16a + 4b = 40

Dividing by 4, we get

4a + b = 10 …(ii)

Solving (i) and (ii), we get

a = 3, b = –2

Miscellaneous Exercise 2 | Q 8 | Page 32

If f(x) = $\dfrac{2x-1}{5x-2}, x \neq \dfrac{2}{5}$

Verify whether (fof) (x) = x

SOLUTION

(fof) (x) = f(f(x))

$$= f\left(\frac{2x-1}{5x-2}\right)$$

$$= \frac{2\frac{2x-1}{5x-2}-1}{5\frac{2x-1}{5x-2}-2}$$

$$= \frac{4x-2-5x+2}{10x-5-10x+4} = \frac{-x}{-1} = x$$

Miscellaneous Exercise 2 | Q 9 | Page 32

If f(x) = $\frac{x+3}{4x-5}$, g(x) = $\frac{3+5x}{4x-1}$ then verify that (fog) (x) = x.

SOLUTION

f(x) = $\frac{x+3}{4x-5}$, g(x) = $\frac{3+5x}{4x-1}$

(fog)(x) = f(g(x))

$$= f\left(\frac{3+5x}{4x-1}\right)$$

$$= \frac{\frac{3+5x}{4x-1}+3}{\left(4\frac{3+5x}{4x-1}\right)-5}$$

$$= \frac{3+5x+12x-3}{12+20x-20x+5} = \frac{17x}{17} = x$$